A Queueing-Theoretic Foundation for Optimal Spaced Repetition

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Motivation

- Spaced repetition software is becoming popular ($\sim 100,000$ users)
- Need models and formalizations that allow us to reason about their operation

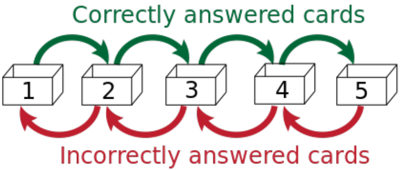
Outline

What is spaced repetition?

- Periodic, spaced review improves long-term retention
- Flashcard software uses spaced repetition
- **Key challenge**: review scheduling (which item should the user review at any given time?)
- **Goal**: ?
- **Constraints**: ?

Heuristic approach

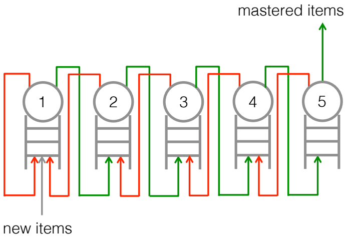
- **Leitner system**: review forgotten items more frequently than recalled items [Leitner, 1974]



- Very little formal understanding of how or why it should work
- Simple theoretical framework proposed by [Novikoff et al., 2012]

Formalizing the Leitner system using queueing theory

- Leitner Queue Network



- New items arrive over time. Mastered items exit the system.
- **Goal**: maximize throughput (the rate at which new items are mastered)
- **Constraints**: user has a review frequency budget (only has time to review U items per day)

Maximizing the rate of learning

deck review rates

throughput

$$\underset{\{\mu_k\}_{k=1}^{n}}{\text{Maximize}} \quad \lambda_{ext} \tag{1}$$

budget constraint

Subject to $U \geq \lambda_{ext} + \sum_{k=1}^{n} \mu_k,$

flow-balance constraint

$$\lambda_1 = \lambda_{ext} + (1 - P_1)\lambda_1 + (1 - P_2)\lambda_2,$$

$$\lambda_k = P_{k-1}\lambda_{k-1} + (1 - P_{k+1})\lambda_{k+1}, \text{ for } k \neq 1, n,$$

$$\lambda_n = P_{n-1}\lambda_{n-1},$$

stability constraint

$$0 \leq \lambda_k \leq \mu_k \quad \forall k \in [n],$$

routing probability

$$P_k = \frac{\mu_k - \lambda_k}{\mu_k - \lambda_k + \theta/k} \quad \forall k \in [n],$$

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Can we model human memory?

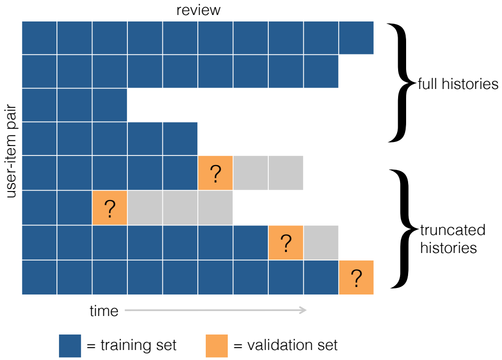
- Assume recall is binary. We would like to **predict the probability of recall**.
- Ebbinghaus proposed the **exponential forgetting curve** in the 19th century [Ebbinghaus, 1913]:

$$\mathbb{P}[\mathrm{recall}] = \exp\left(-\theta \cdot \frac{D}{s}\right),$$

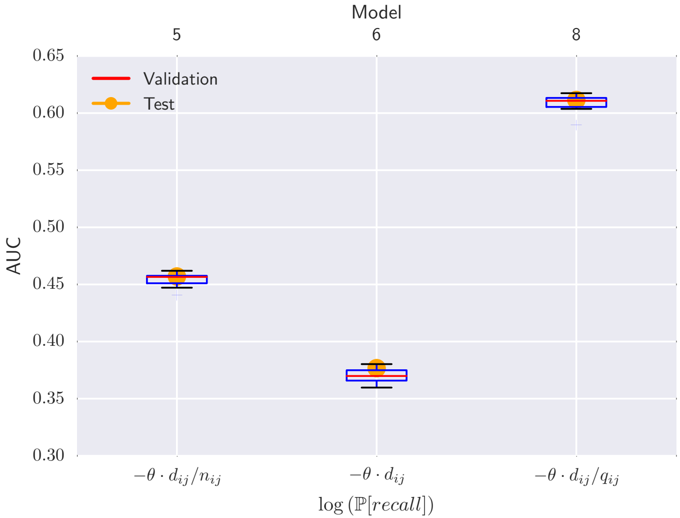
  where θ is the item difficulty, D is the delay since previous review, and s is the memory strength
- Memory strength s is not fully understood

Can we evaluate memory models?

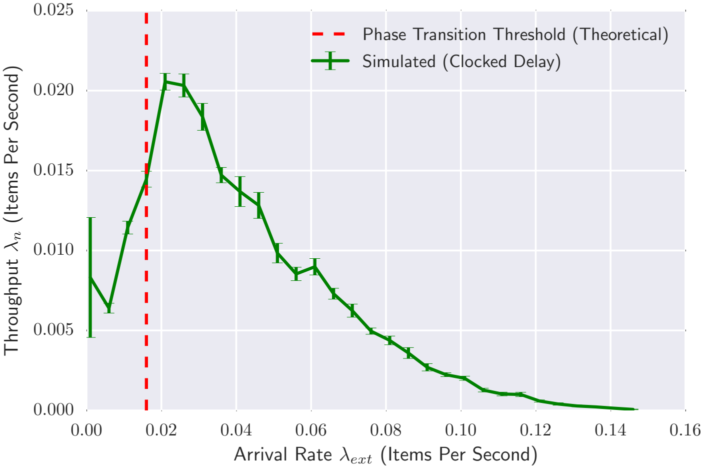
- Large-scale log data from Mnemosyne spaced repetition software [Bienstman, 2006]
- **Prediction task**: given historical outcomes, will the user recall or forget an item?



Results of memory model evaluations



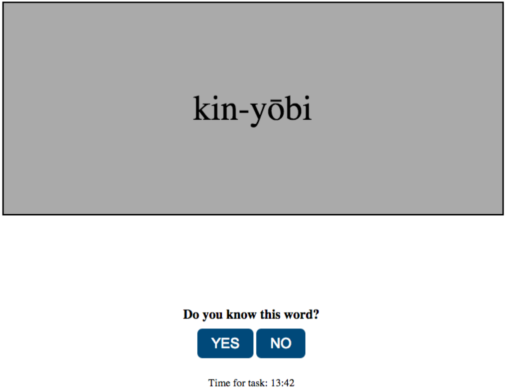
Simulations



Controlled Experiment

- $>$ 300 users on Mechanical Turk [mtu, 2005]
- 15-minute sessions, working through flashcards on Japanese and Ameslan vocabulary
- Users are subjected to different arrival rates of new items
- Can we verify the existence of the phase transition predicted by the Leitner Queue Network?

Controlled Experiment



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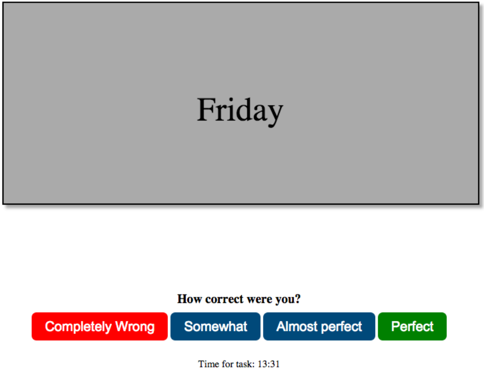

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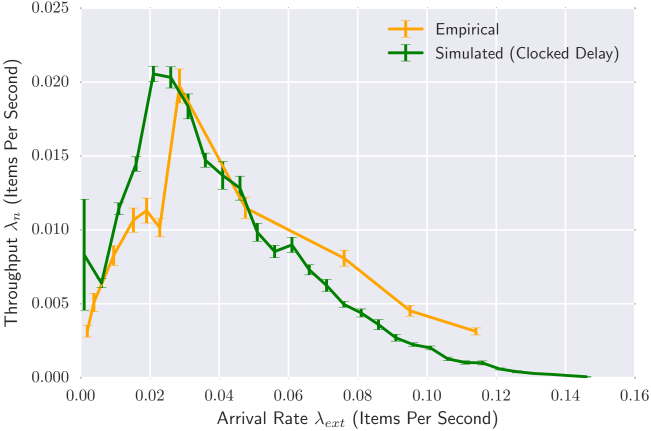

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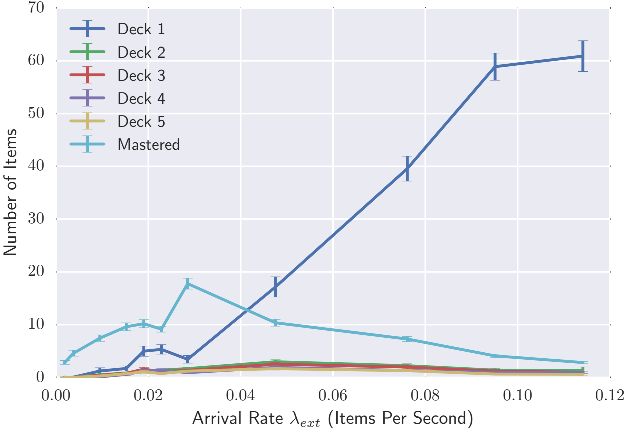

Punch line

Don't procrastinate!

...but if you do, slow down your incoming workload

Ongoing work

Extensions to the Leitner Queue Network:

- Better approximations for routing probabilities
- More sophisticated memory models [Lindsey, 2014]
- Control policies for the transient regime
- Interventions for preventing positive-feedback instabilities

Contact:

- **Email**: sgr45@cornell.edu
- **Paper, Code, Data**: `http://siddharth.io/leitnerq`

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