Optimal Review Scheduling for Human Learning

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Key Contributions

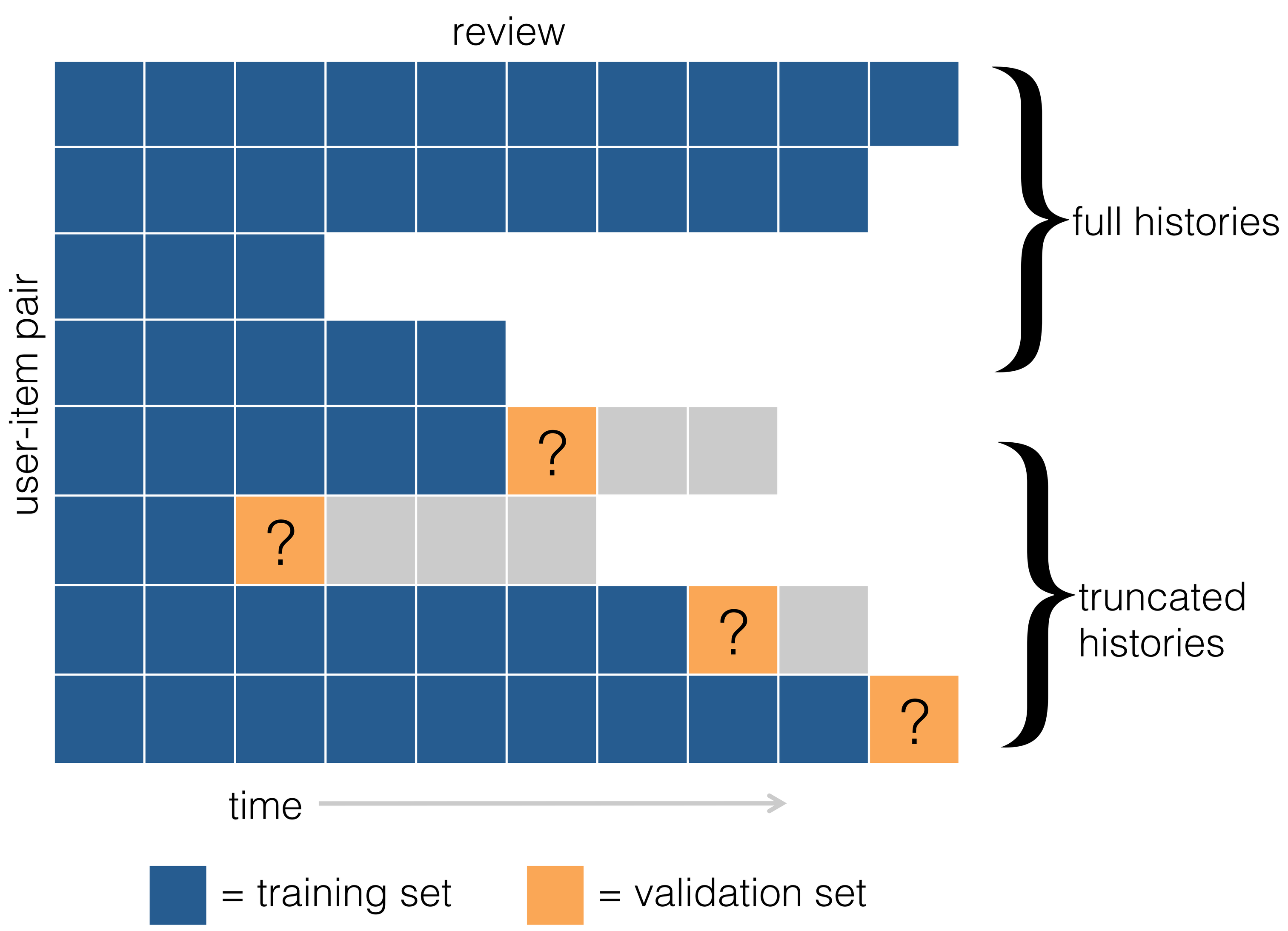
- Evaluation of **human memory models** on log data from **Mnemosyne** project
- New mathematical framework for designing spaced repetition systems; Formalize the **Leitner system** – an algorithm for human memorization of flashcards – using ideas from queueing theory
- Validation of our model via experiments on **Mechanical Turk**

Validating Human Memory Models

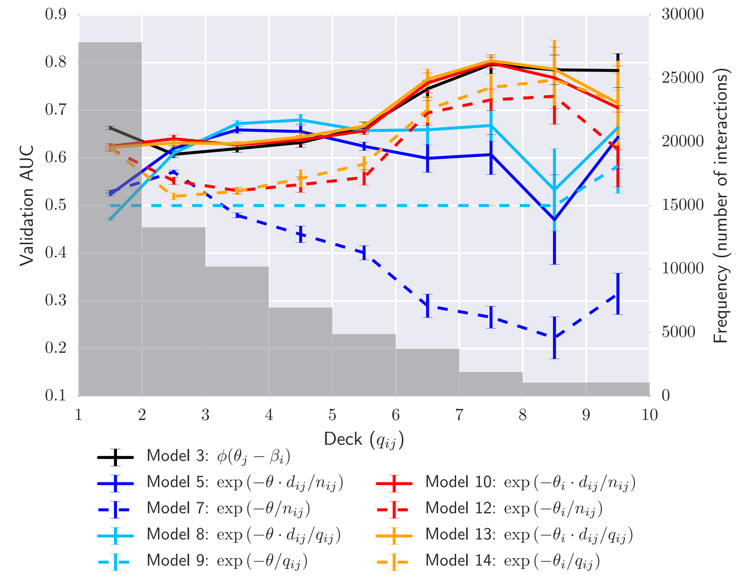
- Assume the **exponential forgetting curve**, which predicts **binary recall**:

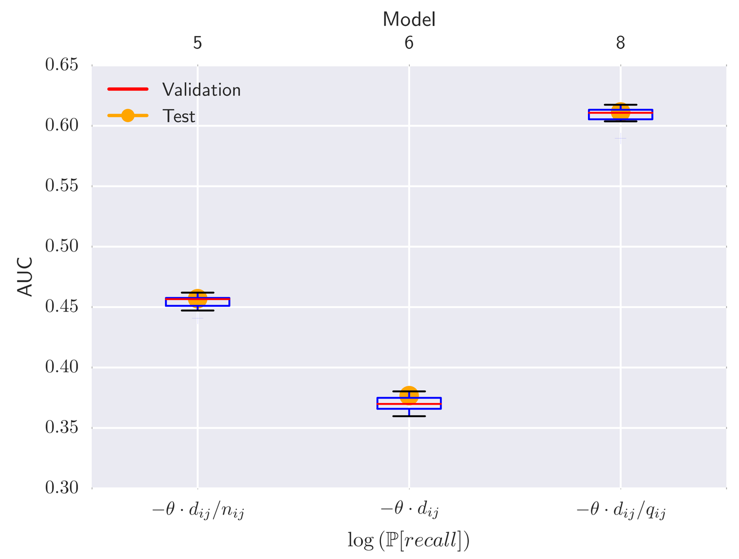
$$\text{Probability of recall} = \exp\left(-\text{item difficulty} \cdot \frac{\text{delay since previous review}}{\text{memory strength}}\right)$$

- Evaluate model on **binary classification task**: predict held-out outcomes in log data from the Mnemosyne spaced repetition software `http://mnemosyne-proj.org`
- Random sample of logs: $\mathbf{859,591}$ **interactions**, $2,742$ users, $88,892$ items, overall recall rate of 0.56

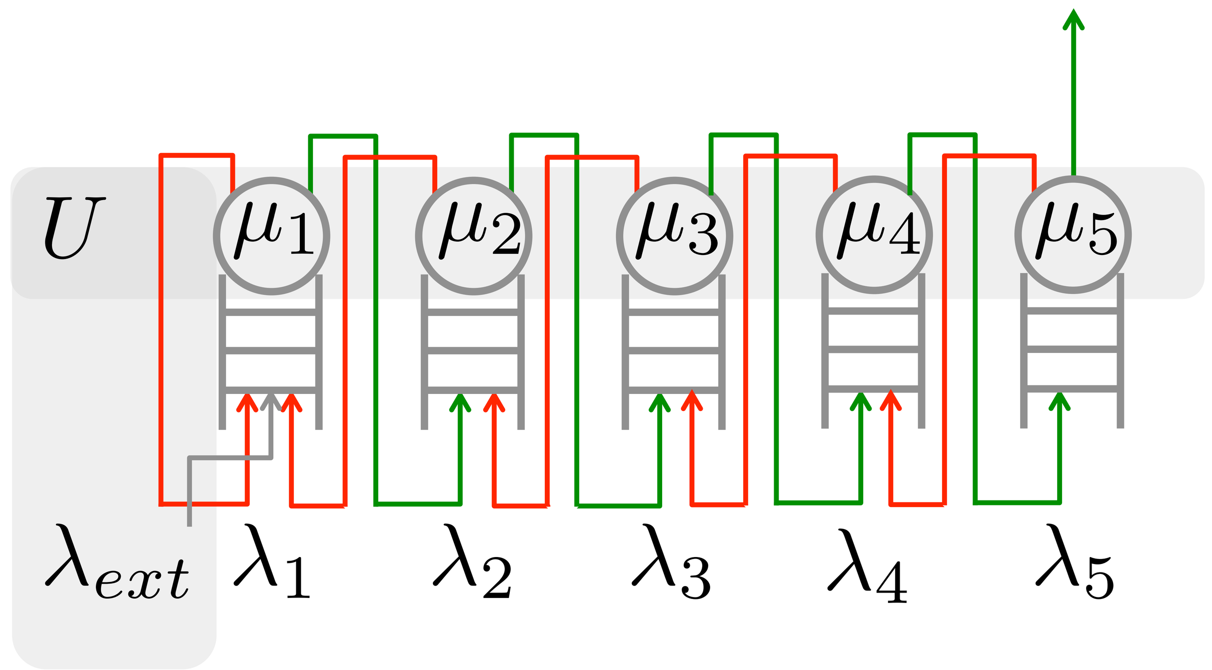


	Probability of recall	Item difficulty	Delay	Strength	Model
3	ϕ(user ability – item difficulty)				1PL item response theory
5	$\exp(-\theta \cdot d_{ij}/n_{ij})$	Global	Time elapsed	Number of reviews	Exp. forgetting curve
6	$\exp(-\theta \cdot d_{ij})$	Global	Time elapsed	Constant	
7	$\exp(-\theta/n_{ij})$	Global	Constant	Number of reviews	
8	$\exp(-\theta \cdot d_{ij}/q_{ij})$	Global	Time elapsed	Leitner deck	
9	$\exp(-\theta/q_{ij})$	Global	Constant	Leitner deck	
10	$\exp(-\theta_i \cdot d_{ij}/n_{ij})$	Item-specific	Time elapsed	Number of reviews	
11	$\exp(-\theta_i \cdot d_{ij})$	Item-specific	Time elapsed	Constant	
12	$\exp(-\theta_i/n_{ij})$	Item-specific	Constant	Number of reviews	
13	$\exp(-\theta_i \cdot d_{ij}/q_{ij})$	Item-specific	Time elapsed	Leitner deck	
14	$\exp(-\theta_i/q_{ij})$	Item-specific	Constant	Leitner deck	





Leitner Queue Network



Mean-recall approximation → tractable **static planning problem** for designing review schedules:

deck review rates — throughput — budget constraint

$$\underset{\{\mu_k\}_{k=1}^n}{\text{Maximize}} \quad \lambda_{ext}$$

$$\text{Subject to } U \geq \lambda_{ext} + \sum_{k=1}^{n} \mu_k,$$

flow-balance constraint:

$$\lambda_1 = \lambda_{ext} + (1 - P_1)\lambda_1 + (1 - P_2)\lambda_2,$$
$$\lambda_k = P_{k-1}\lambda_{k-1} + (1 - P_{k+1})\lambda_{k+1} \text{ , for } k \neq 1, n,$$
$$\lambda_n = P_{n-1}\lambda_{n-1},$$

stability constraint:

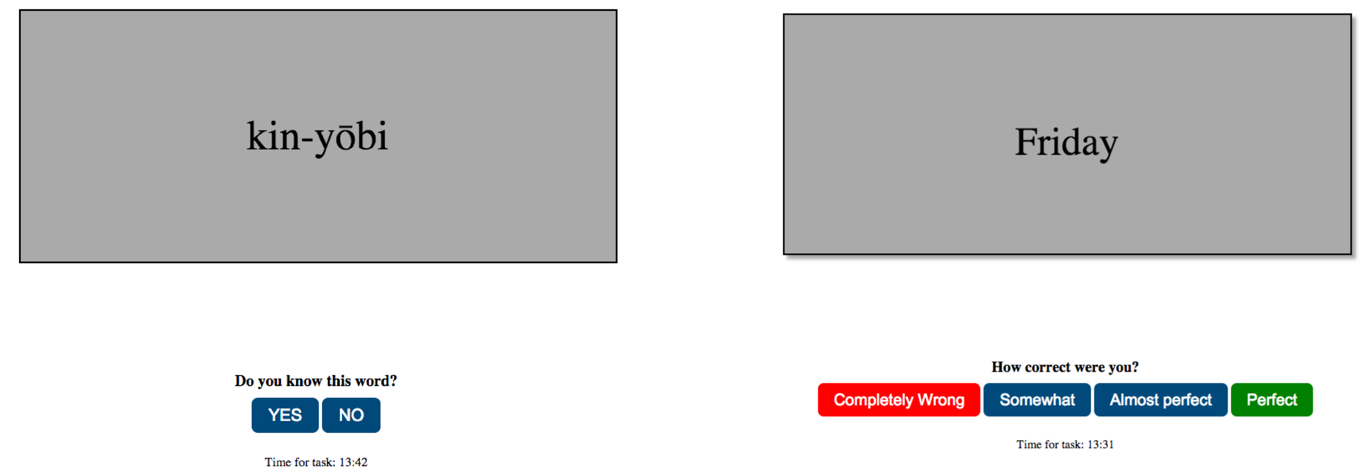
$$0 \leq \lambda_k \leq \mu_k \quad \forall k \in [n],$$

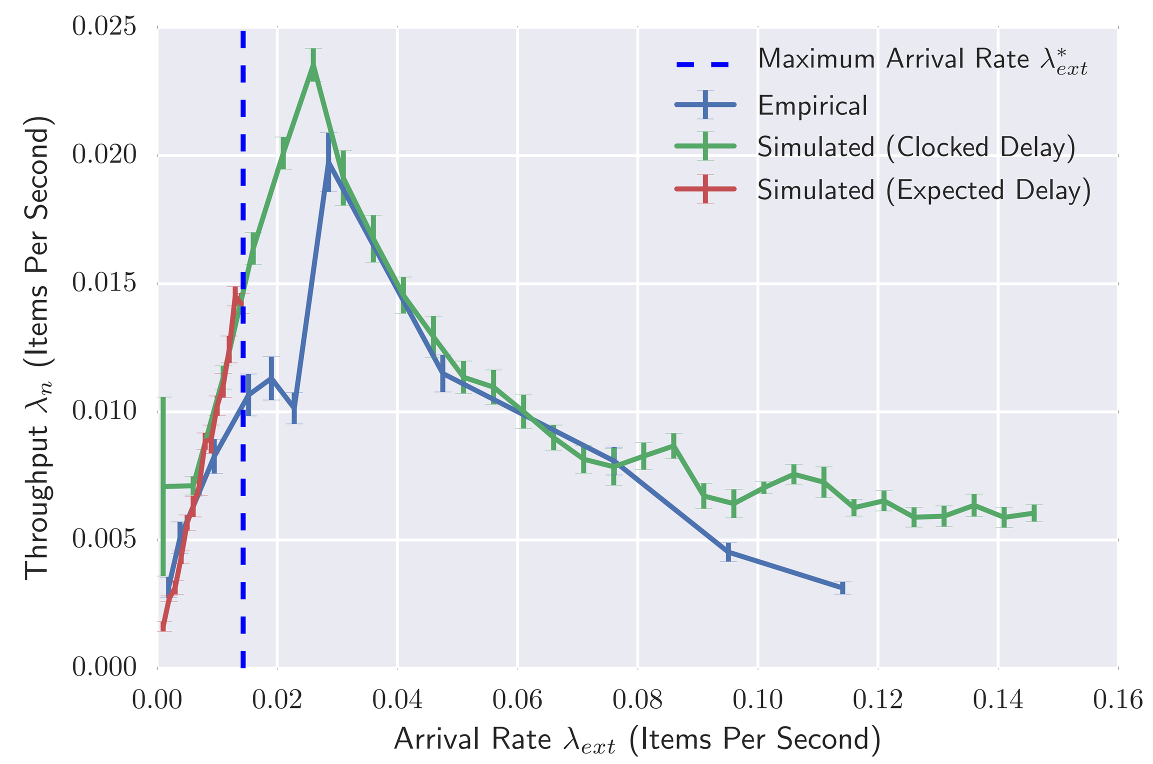
routing probability:

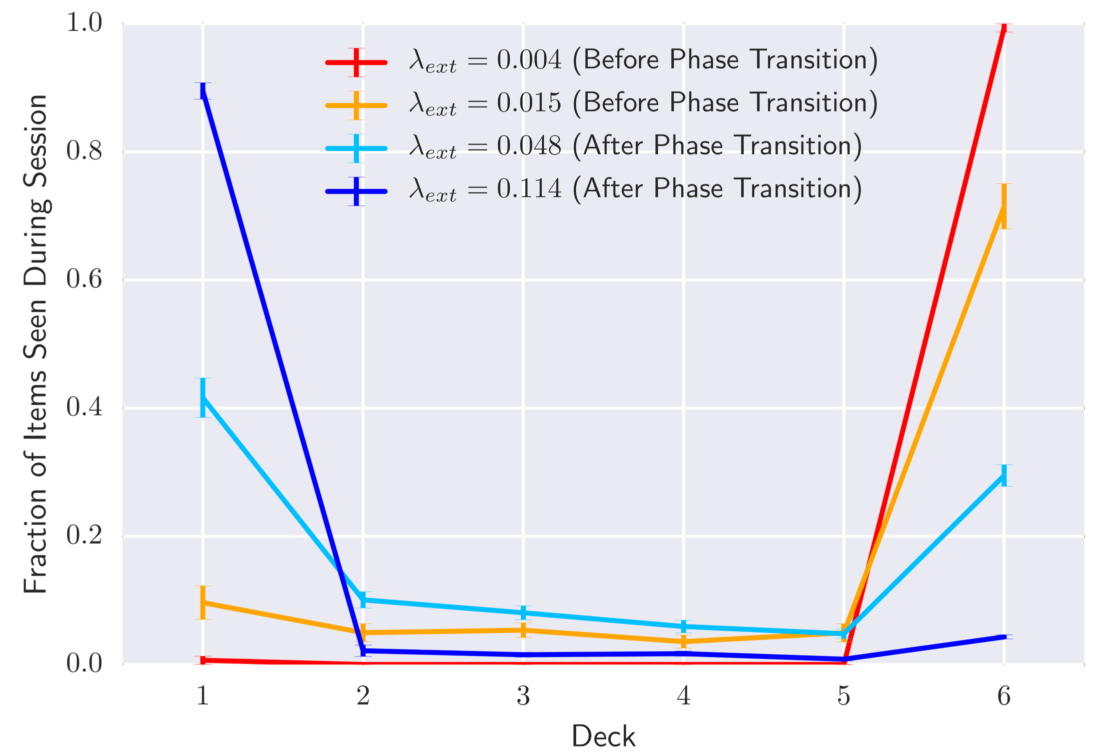
$$P_k = \mathbb{E}_{D_k}\left[\exp\left(-\theta \cdot \frac{D_k}{k}\right)\right] = \frac{\mu_k - \lambda_k}{\mu_k - \lambda_k + \theta/k} \quad \forall k \in [n],$$

Experiments

- **300 turkers** reviewed Japanese and American Sign Language vocabulary flashcards for **15 minutes**
- Observed a **phase transition** in throughput as arrival rate λ_{ext} increased

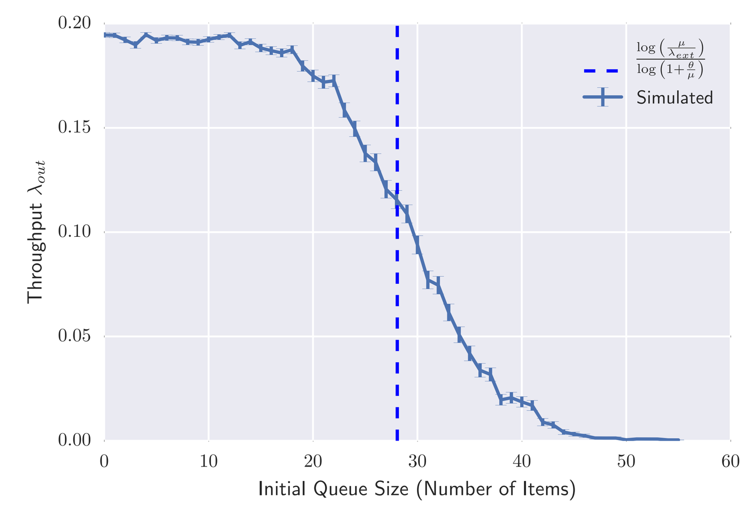


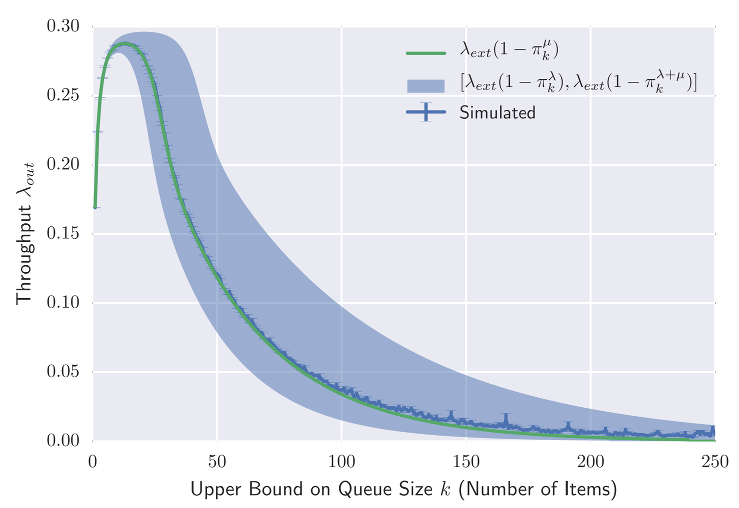




Ongoing Work

- Even for low arrival rates, a **positive feedback loop** can lead to instability
- Intervention: limit the maximum number of items in the system at any given time





Code and data available at `http://siddharth.io/leitnerq`