A Queueing Network Model for Spaced Repetition

Siddharth Reddy, Igor Labutov, Siddhartha Banerjee

{sgr45, iil4, sbanerjee}@cornell.edu

Introduction

The ability to retain a large number of new ideas in memory is an essential component of human learning. In recent times, there has been a growing body of work that attempts to 'engineer' this process – creating tools that enhance the learning process by building on the scientific understanding of human memory. Flashcards are one such tool that use the idea of *spaced repetition* to overcome the human 'forgetting curve'. Though they have been around for a while in the physical form, a new generation of spaced repetition software such as SuperMemo, Anki, and Mnemosyne allow a much greater degree of control and monitoring of the process. As these software applications grow popular, there is a need for formal models for reasoning about and optimizing their operations. In this work, we use ideas from queueing theory to develop such a formal model for one of the simplest and oldest spaced repetition systems: the Leitner system.

Key Contributions

- Formalize the Leitner spaced repetition system using ideas from queueing theory
- Validate the exponential forgetting curve on large-scale log data from the Mnemosyne flashcard software
- Verify key predictions of our queueing model using a controlled experiment on Amazon Mechanical Turk

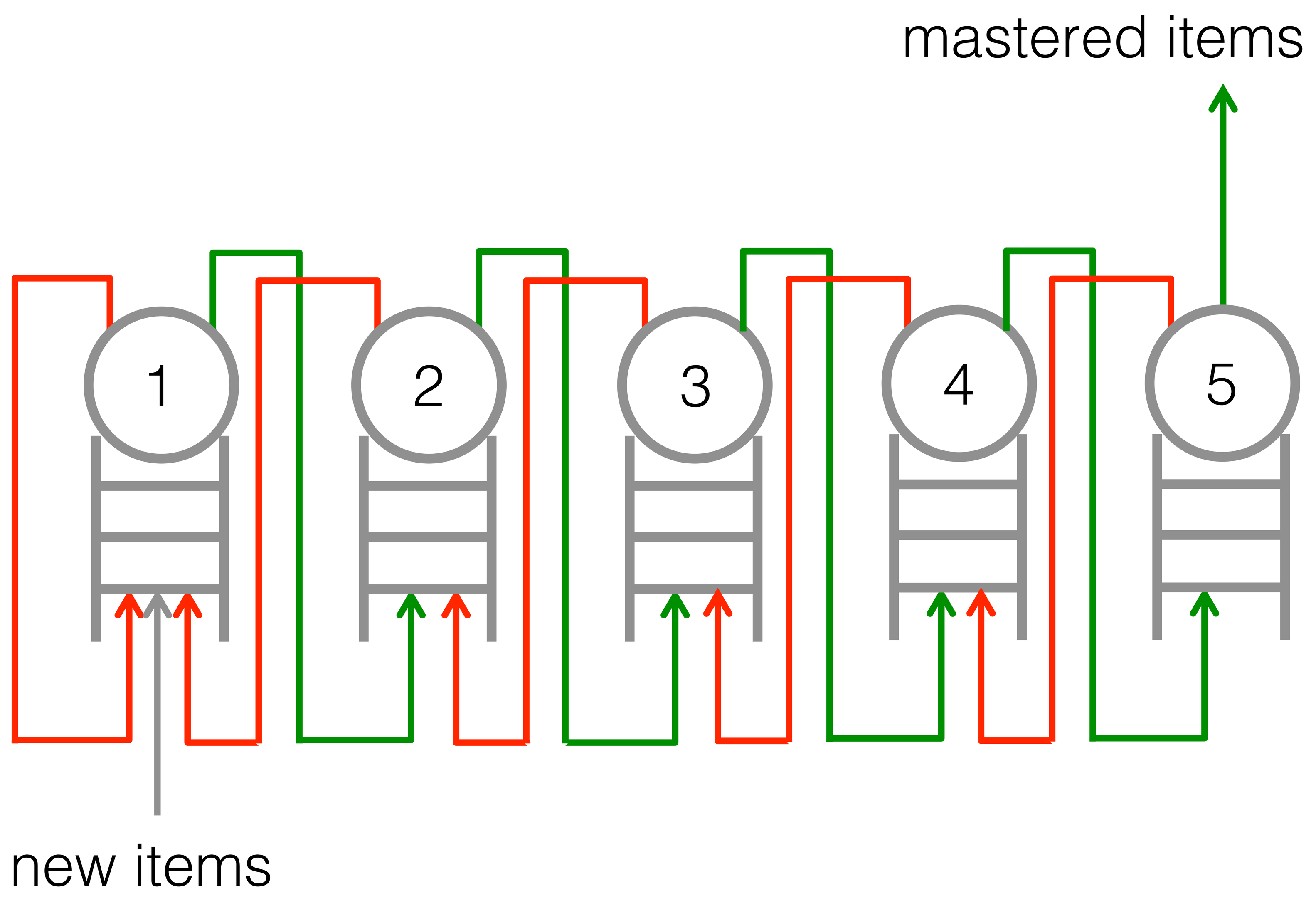
Memory Model

We use a variant on the exponential forgetting curve to model the decay of human memory over time. The probability of a student recalling an item is as follows.

$$\mathbb{P}[\text{recall}] = \exp\left(-\theta \cdot \frac{D}{s}\right)$$

where θ is the item difficulty, D is time elapsed since previous review, and s is memory strength.

Leitner Queue Network



New items arrive into deck 1 according to a Poisson process with rate λ_{ext}. The routing probability matrix is P, where

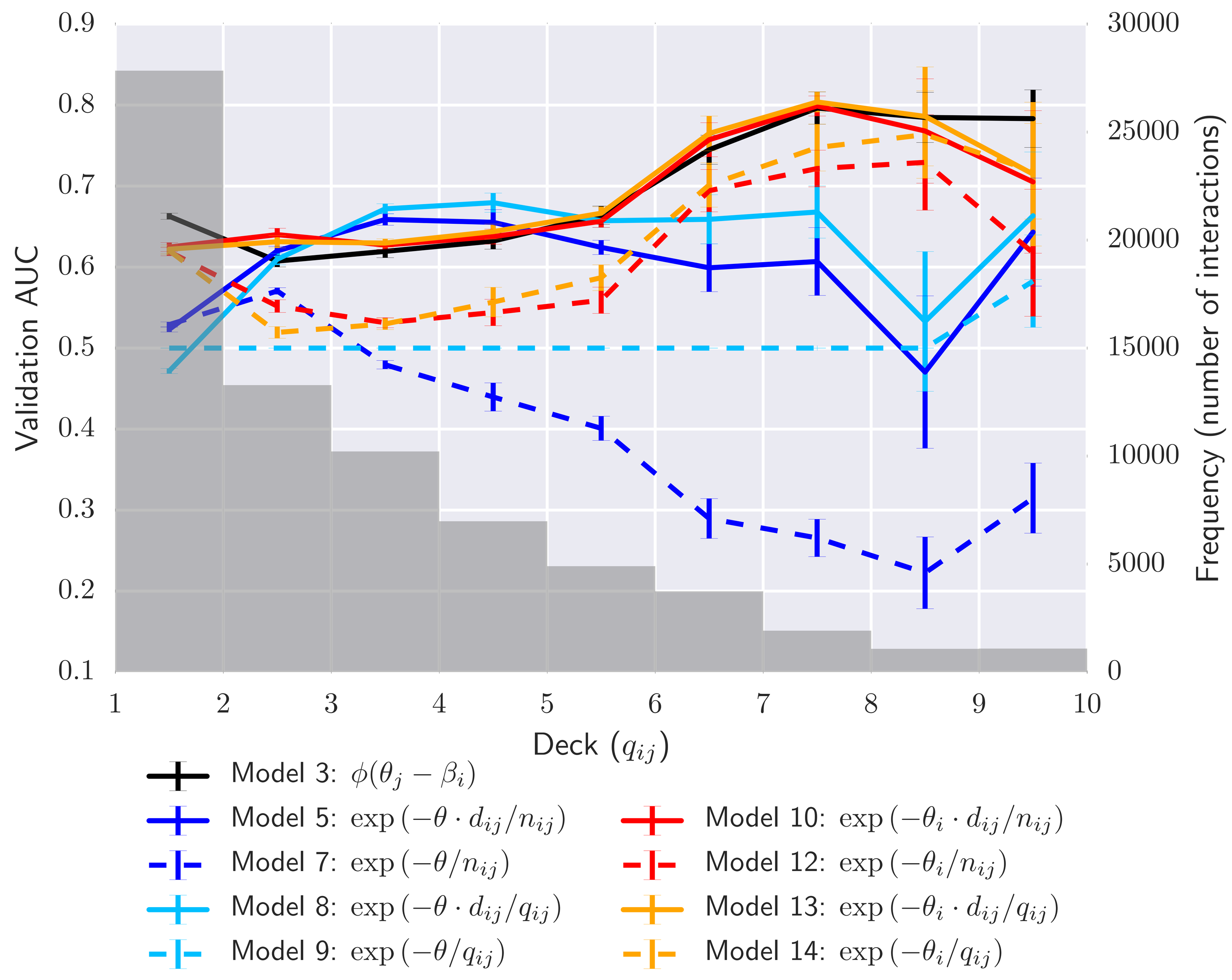
$$P_{ij} = \begin{cases} \mathbb{P}[\text{recall} \mid s = i, \cdot] & \text{if } i < n \wedge j = i + 1 \\ 1 - \mathbb{P}[\text{recall} \mid s = i, \cdot] & \text{if } (i > 1 \wedge j = i - 1) \vee i = j = 1 \\ 0 & \text{otherwise} \end{cases}$$

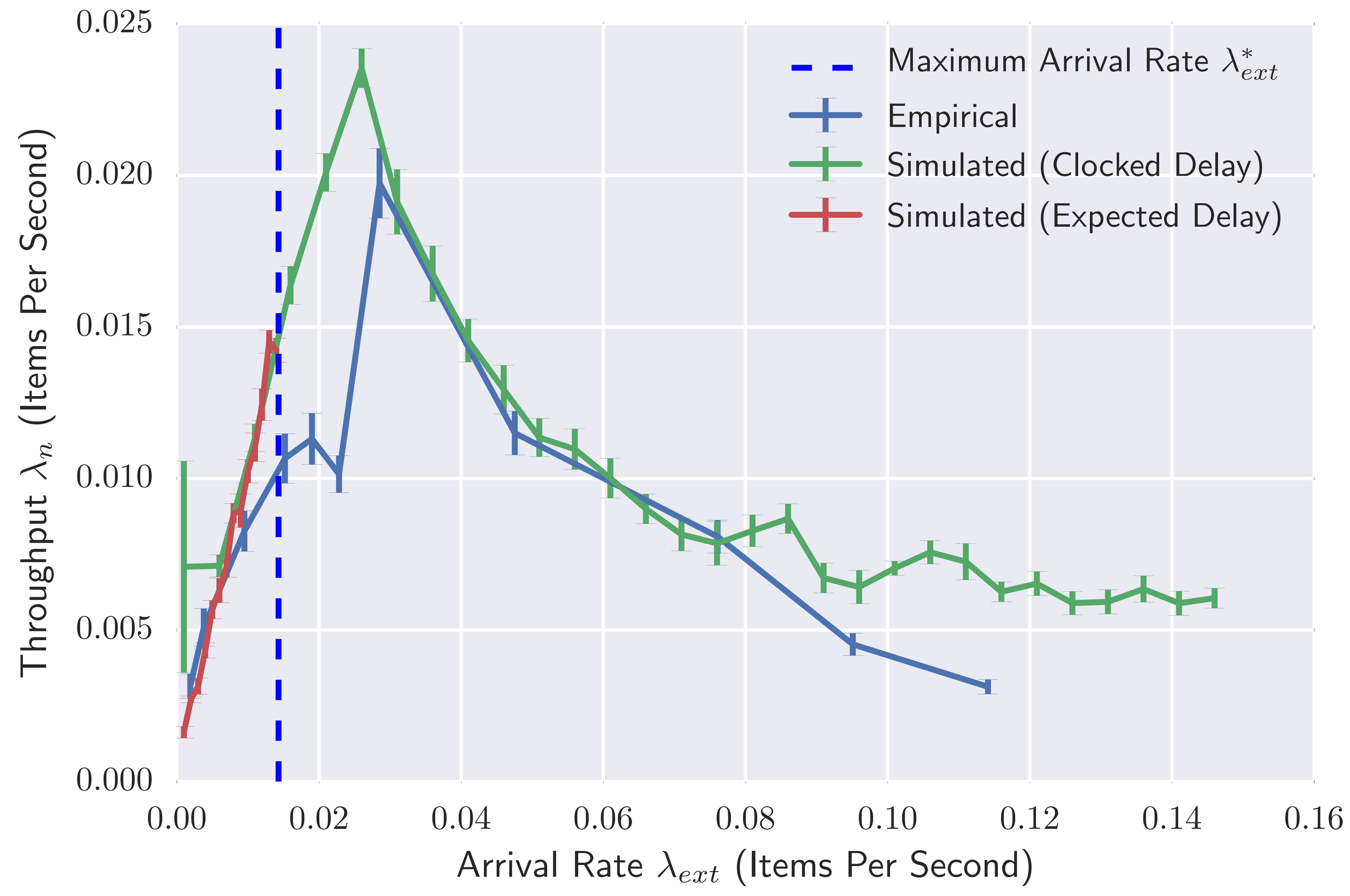
Items exit the system from deck n with probability $\mathbb{P}[\text{recall} \mid s = n, \cdot]$. The service rate for deck i is indicated by μ_i, and the user's work rate budget (e.g., the maximum number of items the user can review per day) is given by U. We are interested in finding μ_i that maximize the steady-state throughput of the system such that the budget constraint $\sum_{i=1}^{n} \mu_i \leq U$ is satisfied. Formally, we must solve the following *static planning problem*.

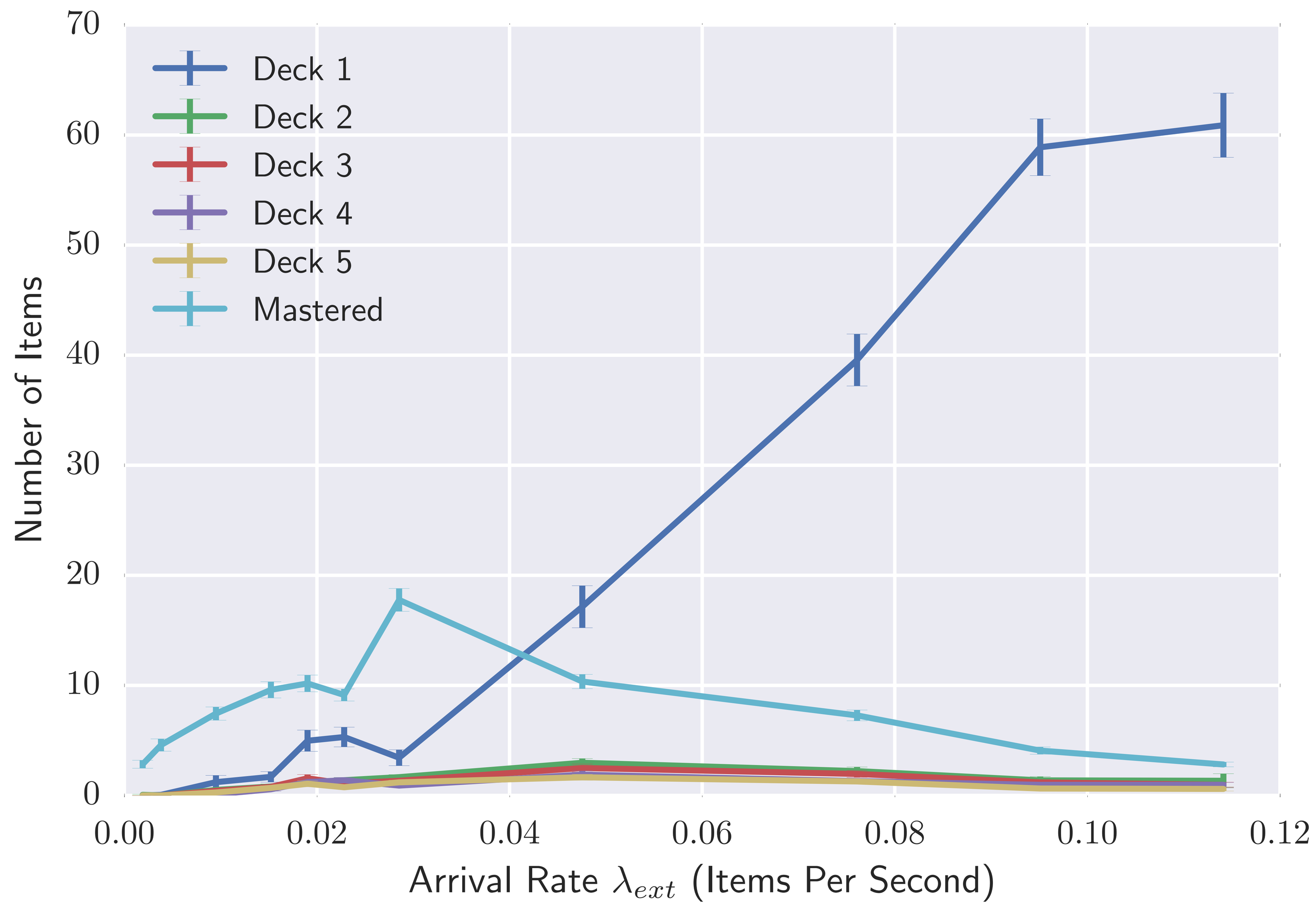
$$\begin{aligned} \text{maximize} \quad & \lambda_{ext} \\ \text{subject to} \quad & \sum_{i=1}^{n} \mu_i \leq U \\ & \lambda_{ext} + P_{11}\lambda_1 + P_{21}\lambda_2 = \lambda_1 \\ & P_{12}\lambda_1 + P_{32}\lambda_3 = \lambda_2 \\ & \vdots \\ & P_{n-1,n}\lambda_{n-1} + P_{nn}\lambda_n = \lambda_n \\ & \mu_i, \lambda_i \geq 0 \qquad \forall i \\ & \lambda_i < \mu_i \qquad \forall i \end{aligned}$$

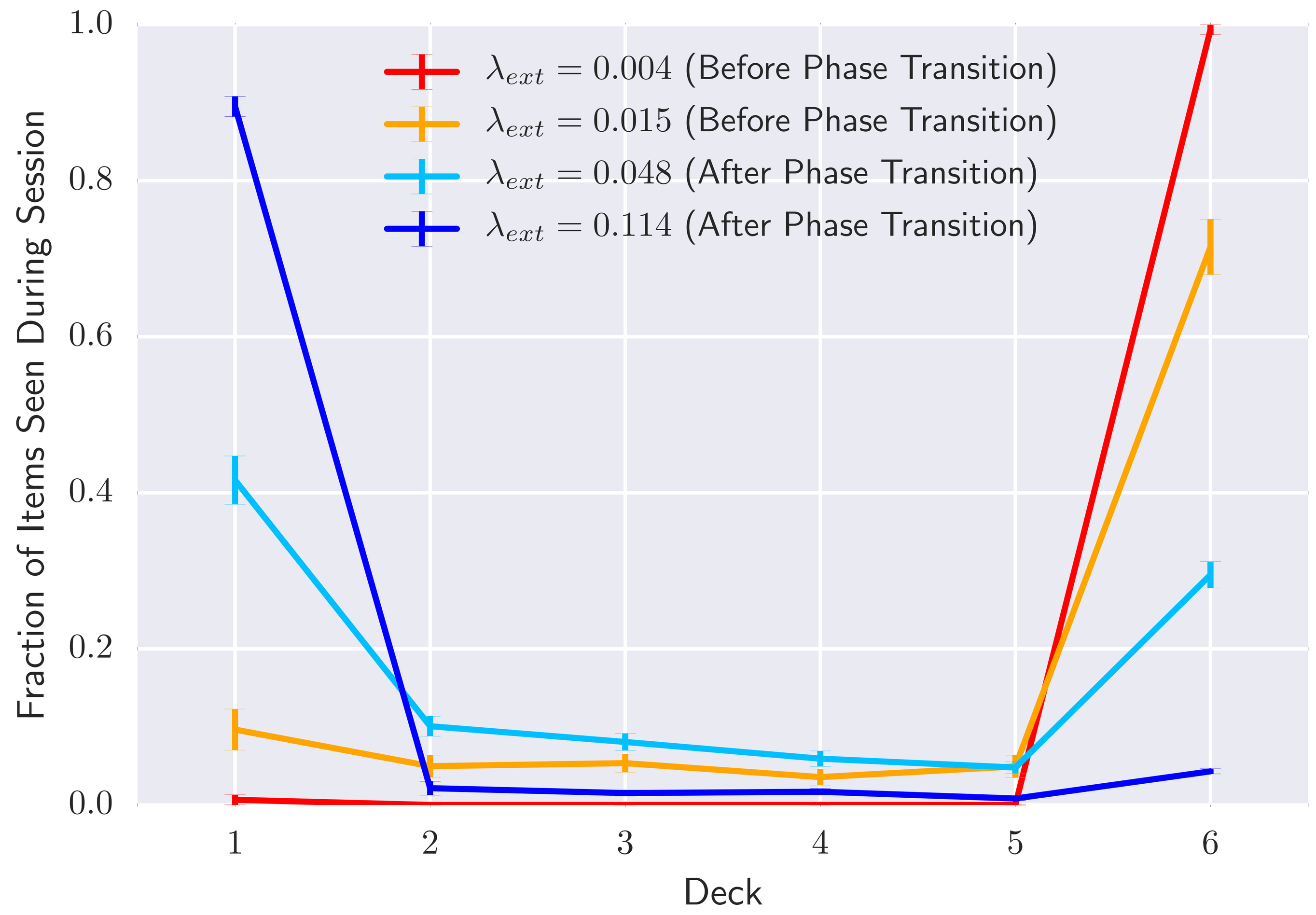
The algorithm for selecting the next item to present to the user is simple: sample deck i with probability $\frac{\mu_i}{\sum_{i=1}^{n} \mu_i}$, and select the item at the top of the sampled deck.

Experiments









Full paper and code available at `http://siddharth.io/leitnerq`